Approach 4

We change $\frac{1}{a}\ln\left|\frac{a+x}{\sqrt{a^2-x^2}}\right|$ to $\frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|$

Step 1:	Step 2:
$\frac{1}{a}\ln\left\|\frac{a+x}{\sqrt{a^2-x^2}}\right\|$	$\frac{1}{a}\left\{\frac{1}{2}\ln\|a+x\|-\frac{1}{2}\ln\|(a-x)\|\right\}$
$\frac{1}{a}\ln\left\|\frac{a+x}{\sqrt{(a+x)(a-x)}}\right\|$	$\frac{1}{a}\bullet\frac{1}{2}\left\{\ln\|a+x\|-\ln\|(a-x)\|\right\}$
$\frac{1}{a}\left\{\ln\|a+x\|-\ln\left\|\sqrt{(a+x)(a-x)}\right\|\right\}$	$\frac{1}{2a}\left\{\ln\|a+x\|-\ln\|(a-x)\|\right\}$
$\frac{1}{a}\left\{\ln\|a+x\|-\ln\left\|\sqrt{(a+x)}\right\|-\ln\left\|\sqrt{(a-x)}\right\|\right\}$	$\frac{1}{2a}\left\{\ln\left\|\frac{a+x}{a-x}\right\|\right\}$
$\frac{1}{a}\left\{\ln\|a+x\|-\ln\left\|(a+x)^{\frac{1}{2}}\right\|-\ln\left\|(a-x)^{\frac{1}{2}}\right\|\right\}$	$\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|$
$\frac{1}{a}\left\{\ln\|a+x\|-\frac{1}{2}\ln\|(a+x)\|-\frac{1}{2}\ln\|(a-x)\|\right\}$	

Lesson 41 Exercises

1. Find $\int\frac{dx}{1+x^2}$

2. Find $\int\frac{1}{1+4x^2}\,dx$

3. Find $\int\frac{dx}{a^2+x^2}$

4. Find $\int\frac{dx}{(1+x^2)^2}$

5. Find $\int\frac{x}{x^2-6x+13}\,dx$

6. Find $\int\frac{x}{4x^2+6x+9}\,dx$

7. Find $\int\frac{dx}{a^2-x^2}$

8. Find $\int\frac{3x^3+5x^2+9x+12}{x^2+4}\,dx$

9. Find $\int\frac{\sqrt{x}}{\sqrt[3]{x}+1}\,dx$

10. $\int\frac{1}{4x^2+6x+9}\,dx$

Answers: **1.** $\text{Tan}^{-1}x+C$; **2,** $\frac{1}{2}\text{Tan}^{-1}2x+C$; **3.** $\frac{1}{a}\text{Tan}^{-1}\frac{x}{a}+C$

4. $\frac{x}{2(1+x^2)}+\frac{1}{2}\arctan x+C$; **5.** $\frac{1}{2}\ln\left|(x-3)^2+4\right|+\frac{3}{2}\tan^{-1}\left(\frac{x-3}{2}\right)+C$

6. $=\frac{1}{8}\left[\ln\left|(4x^2+6x+9\right|\right]-\frac{\sqrt{3}}{12}\arctan\frac{\sqrt{3}}{9}(4x+3)+C$;

7. $\frac{1}{a}\ln\left|\frac{a+x}{\sqrt{a^2-x^2}}\right|+C$; or $\frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+C$

8. $\frac{3}{2}x^2+5x-\frac{3}{2}\ln\left|x^2+4\right|-4\text{Tan}^{-1}\left(\frac{x}{2}\right)+C$

9. $\frac{6}{7}x^{\frac{7}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2x^{\frac{1}{2}}-6x^{\frac{1}{6}}+6\text{Tan}^{-1}(x^{\frac{1}{6}}))+C$;

10. $=\frac{\sqrt{3}}{9}\arctan\left[\frac{\sqrt{3}}{9}(4x+3)\right]+C$.