Lesson 32

Integration of Radical Functions I

Classification of Integration of Radical Functions
We may classify the integration of radical functions by antiderivative type, by integrand type, or by method type. For some functions ,we will use simple u-substitution, and for others we will use trigonometric substitution. In this lesson, we use only simple u-substitution. In Lesson 43, Integration of Radical Functions II, we use trigonometric substitution. Before proceeding, review Lesson 30.

Integration of Radical Functions Using Simple U-Substitution

Examples **1.** $\int \sqrt{x-5}\, dx$; **2.** $\int \frac{4x^2}{\sqrt{x^3+1}}\, dx$; **3.** $\int 4x^2\sqrt{x^3+1}\, dx$.

Note: 1 is composite; **2** and **3** contain composite functions.

Example 1 Find $\int \sqrt{x-5}\, dx$

Solution We use simple **u-substitution** with **two approaches**

Approach 1

$\int \sqrt{x-5}\, dx$

Let $u = x-5$

Then $\frac{du}{dx} = 1$, and $du = dx$

Substituting for $x-5$ and dx in

$\int \sqrt{x-5}\, dx$, we obtain

$\int \sqrt{u}\, du$

$= \int u^{\frac{1}{2}} du$

$= \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$= \frac{2}{3}u^{\frac{3}{2}} + C$

$= \frac{2}{3}(x-5)^{\frac{3}{2}} + C$ (back to x).

$= \frac{2}{3}\left(\sqrt{x-5}\right)^3 + C$. or

$= \frac{2}{3}(x-5)\sqrt{x-5} + C$

Approach 2

Let $u = \sqrt{x-5}$ (A)

Then $u^2 = x-5$

Using implicit differentiation,

$2u\frac{du}{dx} = 1$, and $2u\, du = dx$

Substituting for $\sqrt{x-5}$ and dx in

$\int \sqrt{x-5}\, dx$, we obtain

$\int u \bullet 2u\, du$

$= \int 2u^2 du$

$= 2\int u^2 du$

$= 2 \bullet \frac{u^3}{3} + C$

$= \frac{2}{3}\left(\sqrt{x-5}\right)^3 + C$ (back to x). or

$= \frac{2}{3}(x-5)\sqrt{x-5} + C$