Simple Direct Proportion (Direct Variation)

Definition: A **proportion** is a statement that two ratios are equal.
The direct proportion "*a* is to *b* as *c* is to *d* " (or *a*:*b*::*c*:*d*) is the equality

$$\frac{a}{b} = \frac{c}{d} \qquad (\textbf{quotient = quotient})$$

In the above proportion, a and d are called the extremes and b and c are called the means of the proportion. We may also define a proportion as a mathematical statement indicating the equality between two **equivalent fractions.** Example: $\frac{2}{3} = \frac{4}{6}$ is a proportion

Proportion Principles

1. We can invert both ratios of a proportion: From (1) above, we obtain after the inversion,

$$\frac{b}{a} = \frac{d}{c}$$

2. We can interchange the extremes of a proportion: From (1) above we would obtain:

$$\frac{d}{b} = \frac{c}{a}$$

3. We can also interchange the means to obtain:

$$\frac{a}{c} = \frac{b}{d}$$

Also $ad = bc$ (That is the product of the extremes = the product of the means) <-- also called cross-multiplication

The proportion principles can be used to obtain desired ratios. For example, if we have $\frac{3}{2}$ as the ratio of one side of a proportion, and we desire the ratio $\frac{2}{3}$ then proportion principle #1 above (inversion of both sides of a proportion) can be used to obtain this ratio, $\frac{2}{3}$, from $\frac{3}{2}$.

More Principles (From $\frac{a}{b} = \frac{c}{d}$**,** we obtain the following:

4. $\frac{a+b}{b} = \frac{c+d}{d}$ (by addition). Note: $\frac{a+b}{b} = \frac{a}{b} + \frac{b}{b} = \frac{a}{b} + 1$ **and** $\frac{c+d}{d} = \frac{c}{d} + \frac{d}{d} = \frac{c}{d} + 1$

5. $\frac{a-b}{b} = \frac{c-d}{d}$ (by subtraction)

6. $\frac{a-b}{a+b} = \frac{c-d}{c+d}$ (by subtraction and addition)

7. $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (by addition and subtraction)

8. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a+c+e}{b+d+f} = \frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

e.g.,($\frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{2+3+4}{10+15+20} = \frac{9}{45} = \frac{1}{5}$)

To show that **8** is true, see Appendix, p.566

Problems Involving Proportion

Example 1 Find x from the proportion x is to 4 as 3 is to 2.

Solution $\frac{x}{4} = \frac{3}{2}$

$2x = 12$ (cross multiplication)

$x = 6$

(You could also solve the above equation by multiplying both sides of the equation by 4)

Example 2 Find x from the proportion: 6 is to x as 4 is to 12.

Solution $\frac{6}{x} = \frac{4}{12}$

$4x = 72$; and $x = 18$

To show that if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$

We proceed in two steps. In Step 1, we consider $\frac{a}{b} = \frac{c}{d}$. In Step 2, We consider the result from Step 1 and $\frac{e}{f}$.

Step 1: $\frac{a}{b} = \frac{c}{d}$ (Given)

$\frac{a}{b} + \boxed{\frac{c}{b}} = \frac{c}{d} + \boxed{\frac{c}{b}}$ (Adding $\frac{c}{b}$ to both sides of the equation)

$\frac{a+c}{b} = \frac{bc+cd}{bd}$ (Adding on the LHS and on the RHS of the equation)

$\frac{a+c}{b} = \frac{c(b+d)}{bd}$ (Factoring the RHS)

$\frac{a+c}{b+d} = \frac{bc}{bd}$ (Dividing both sides by $b+d$ and multiplying both sides by b)

$\frac{a+c}{b+d} = \frac{c}{d}$ (Dividing out the b on the RHS)

Step 2: $\frac{a+c}{b+d} = \frac{e}{f}$ (Since $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, replace $\frac{c}{d}$ on the RHS by $\frac{e}{f}$)

$\frac{a+c}{b+d} + \boxed{\frac{e}{b+d}} = \frac{e}{f} + \boxed{\frac{e}{b+d}}$ (Adding $\frac{e}{b+d}$ to both sides of the equation)

$\frac{a+c+e}{b+d} = \frac{e(b+d)+ef}{f(b+d)}$ (Adding on the LHS and on the RHS of the equation)

$\frac{a+c+e}{b+d} = \frac{be+de+ef}{f(b+d)}$

$\frac{a+c+e}{b+d} = \frac{e(b+d+f)}{f(b+d)}$ (Factoring out the e on the RHS)

$\frac{a+c+e}{b+d+f} = \frac{e(b+d)}{f(b+d)}$ (Dividing both sides by $b+d+f$ and multiplying by $b+d$)

$\frac{a+c+e}{b+d+f} = \frac{e}{f}$ (Dividing out the $b+d$ on the RHS)

Since $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, (given)

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$

Application of the above to similar triangles.
Consider two similar triangles ABC and DEF.
If the lengths of the sides of ΔABC are a, b, c and the corresponding lengths of the sides of Δ DEF are d, e, f, then for the ratios of corresponding sides, $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$ and

$\frac{a}{d} = \frac{b}{e} = \frac{c}{f} = \frac{a+b+c}{d+e+f}$. Thus $\frac{a}{d} = \frac{b}{e} = \frac{c}{f} = \frac{\text{perimeter of } \Delta \text{ ABC}}{\text{perimeter of } \Delta \text{ DEF}}$

Example Let the lengths of sides of Δ ABC be 2, 3, 4; and let the corresponding lengths of the sides Δ DEF be 10, 15. 20,. Then $\frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \frac{2+3+4}{10+15+20} = \frac{9}{45} = \frac{1}{5}$