Note: The inverse functions $\text{Csc}^{-1}x$, $\text{Sec}^{-1}x$, and $\text{Cot}^{-1}x$ can be expressed in terms of $\text{Sin}^{-1}x$, $\text{Cos}^{-1}x$, and $\text{Tan}^{-1}x$ respectively as follows:

$\text{Csc}^{-1}x = \text{Sin}^{-1}\left(\frac{1}{x}\right)$, $\text{Sec}^{-1}x = \text{Cos}^{-1}\left(\frac{1}{x}\right)$, and $\text{Cot}^{-1}x = \text{Tan}^{-1}\left(\frac{1}{x}\right)$.

For example, $\text{Csc}^{-1}2 = \text{Sin}^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$. That is, to find $\text{Csc}^{-1}2$, first find the reciprocal of 2, which is $\frac{1}{2}$ and then find $\text{Sin}^{-1}\left(\frac{1}{2}\right)$, which is $\frac{\pi}{6}$. We may therefore memorize the definitions for only $\text{Sin}^{-1}x$, $\text{Cos}^{-1}x$, and $\text{Tan}^{-1}x$ and then apply the above note.

Example 1 Find Arcsin (.866). (Note that Arcsin (.866) is an angle or a number.)

Step 1: Let y = Arcsin (.866) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. (We will find y, the angle whose sine is .866)

Two conditions must be satisfied: 1. $\sin y = .866$); **2**. $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. (same as y is in first or fourth quadrant)

Step 2: Find the reference angle for the angle whose sine .866

From memory or calculator (or tables), the reference angle is $\frac{\pi}{3}$. (Note that $\frac{\pi}{3} = 60°$)

Step 3: Find the quadrant in which the sine function is positive.

The sine function is positive in the **first** and second quadrants.

Since by definition, Arcsin x must be in either the **first** or the fourth quadrant, $(-\frac{\pi}{2} \le y \le \frac{\pi}{2})$,

Arcsin x is in the **first** quadrant where it is a positive angle.

Therefore, Arcsin (.866) = $\frac{\pi}{3}$. (Note that $-\frac{\pi}{2} \le \frac{\pi}{3} \le \frac{\pi}{2}$)

Example 2 Find Arcsin (-.5). (Note that Arcsin (-.5) is an angle or a number.)

Step 1: Let y = Arcsin $(-.5)$ $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. (We will find y, the angle whose sine is -.5)

Step 2: Find the reference angle for the angle whose sine -.5.

From tables or memory (or calculator), the reference angle is $\frac{\pi}{6}$.

Step 3: Find the quadrant in which the sine function is negative.

The sine function is negative in the third and **fourth** quadrants.

Since Arcsin x must be in either the first or the **fourth** quadrant, $(-\frac{\pi}{2} \le y \le \frac{\pi}{2})$,

Arcsin x is in the fourth quadrant where it is a negative angle.

Therefore, Arcsin (-.5) = $-\frac{\pi}{6}$.

Example 3 Find $\text{Cot}^{-1}(-2.7)$ using a calculator.

: Step 1: Find $\frac{1}{2.7} = 0.37037037$ (Summarily, $\text{Cot}^{-1}(-2.7) = \pi - \text{Cot}^{-1}(2.7) = \pi - \text{Tan}^{-1}(\frac{1}{2.7})$)

Step 2: Find $\text{Tan}^{-1}\left(\frac{1}{2.7}\right) = \text{Tan}^{-1}(0.37037037) = 0.35$ <--- Reference angle in radians

The reference angle for $\text{Cot}^{-1}(-2.7) = 0.35$. By definition, when x is negative,

$\text{Cot}^{-1}x$ is in the second quadrant and the terminal side of the angle is in the second quadrant.

Therefore, $\text{Cot}^{-1}(-2.7) = \pi - 0.35$ $\boxed{= 2.79}$.